

B.Sc. Honours Internal Evaluation- 2020

(Under CBCS Pattern)

Semester-II

Subject-Mathematics

Paper-C3T: Real Analysis

Candidates are required to give their answer in their own words as far as practicable. Questions are of equal value.

Full marks-30

Answer any five question

from the following:

1. State and prove Bolzano-Weierstrass theorem.
2. Let $S = \left\{(-1)^m + \frac{1}{n} : m \in N, n \in N\right\}$,
 - i) Show that 1 and -1 are limit points of S.
 - ii) Find the derived set of S.
3. Let F be a closed set in \mathbf{R} . Then the complement of F (in \mathbf{R}) is open in \mathbf{R} .
4. Let A be a non-empty set of \mathbf{R} and $d(x, A) = \inf\{|x - y| : y \in A\}$. Prove that $d(x, A) = 0$, if and only if $x \in \bar{A}$.
5. Is the union of infinite number of closed sets in \mathbf{R} is closed set? Prove or disprove it.
6. Prove that a Sequence can have at most one limit.
7. Prove that a monotone increasing sequence, if bounded above, is convergent and it converges to least upper bound.
8. State and prove the Sandwich theorem. And using this theorem find the value of

$$\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + n}} \right)$$