বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - II

Subject : MATHEMATICS

Paper : C 4-T

Differential Equations & Vector Calculus

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Answer any ***four*** of the following questions : $4 \times 15 = 60$

1. (a) Solve : $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} + y = x^2 \cos x$ 8

   (b) Solve : $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ 7

2. (a) By the method of undetermined co-efficient solve :

   $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x$$ 10

(b) Determine the unit vector perpendicular to the plane of :

$\bar{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\bar{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. 5

3. (a) Find the solution of the equation $\frac{d^2x}{dt^2} - x = 2$, which satisfies the conditions $\frac{dx}{dt} = 3$ when x = 1 and t = 2 when x = –1. 10

(b) Solve the initial value problem $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$, y(0) = 3, $y'(0) = 5$. 5

4. (a) Given that y = x is a solution of $(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, find the linearly independent solution by reducing the order. Write the general solution. 8

(b) Suppose $\bar{A} = x^2yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$ and $\bar{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$.

Find $\frac{\partial^2}{\partial x \partial y}(\bar{A} \times \bar{B})$ at (1, 0, –2) 7

5. (a) With the use of method of variation of parameter solve :

$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{e^{-3x}}{x^3}$ 8

(b) Solve the systems : $2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$

$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$ 7

6. (a) Find the power series solution in power of x of the following differential equation $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$. 10

(b) From the first principle show that $\frac{d}{du}(\bar{B} \times \bar{C}) = \left(\bar{B} \times \frac{d\bar{c}}{du}\right) + \left(\frac{d\bar{B}}{du} \times \bar{C}\right)$ 5

7. Use the method of Frobenious to find solution near $x = 0$ of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$$ 15

8. (a) If $\bar{F}=xy\hat{\imath}-z\hat{\jmath}+x^2\hat{k}$, Evaluate $\int_C \bar{F}\times d\bar{r}$,

where $C : x = t,\ y = 2t,\ z = t^3;\ t:0\to1$ 8

(b) Prove that $\bar{A}\times\left(\bar{B}\times\bar{C}\right)=\bar{B}\left(\bar{A}\cdot\bar{C}\right)-\bar{C}\left(\bar{A}\cdot\bar{B}\right)$. 7