বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examination 2021

(Under CBCS Pattern)

Semester - VI

Subject: MATHEMATICS

Paper: DSE 1B/2B/3B - T

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.
The figures in the margin indicate full marks.

MECHANICS

Answer any ***four*** from the following questions : 15×4=60

1. (a) Find the centre of gravity of the area included between the curve $y^2(2a-x)=x^3$ and its asymptotes.

   (b) Obtain the formula for centre of gravity of a plane area. Find the centre of gravity of a plane lamina of uniform density in the form of a quadrant of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

   7+8

2. (a) A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8\pi - 4}$ from the centre of the circle where a is the diameter of the circular lamina.

(b) A ladder, whose centre of gravity divides in into two portion of length 'a' and 'b', rest with one end on a horizontal floor and the other end against a rough vertical wall. If the coefficienty of friction at the floor and the wall are respectively μ and $\mu^{/}$, show that the inclination of the ladder to the floor, when the equilibrium is limiting, is $\tan^{-1}\frac{a - b\mu\mu}{\mu(a+b)}$. 7+8

3. (a) Prove that the necessary and sufficient conditions that a system of co-planner forces acting on a rigid body be in equilibrium are that the algebraic sum of the resolved parts of the forces in any two mutually perpendicular directions in the plane be separately zero and the algebaric sum of the moments of the forces about any point in their plane should also be zero.

(b) Moments of the resultant R of a system of coplanar forces about three points O, A and B lying in the plane of the forces are $G, G+J_1, G+J_2$. If referred to O as origin the polar co-ordinates of A and B are (r_1, a_1) and (r_2, a_2), show that

$$R^2 \sin^2(\alpha_1 - \alpha_2) = \frac{J_1^2}{r_1^2} + \frac{J_2^2}{r_2^2} - 2\frac{J_1 J_2}{r_1 r_2}\cos(\alpha_1 - \alpha_2).$$ 8+7

4. (a) Stat laws of statistical friction. Define angle of friction and cone of friction. Investigate the condition of equilibrium of a particle constrained to rest on a rought plane curve $f(x, y) = 0$ under any given forces in the plane of curve.

(b) A rough wire which has the shape of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is placed with its x-axis vertical and y axis horizontal, if μ the co-efficient of friction, find the depth below the highest point of the position of limiting equilibrium of a bead which rest on the wire. 8+7

5. (a) A heavy uniform chain, of length $2l$, hangs over, a smooth fixed pulley, the length l+c being at one side and l – c at the other; if it released from a state of rest, show by the principle of energy, that the chain will slip off of the pulley in time $\left(\frac{l}{g}\right)^{\frac{1}{2}} \log_e \frac{1+\sqrt{l^2 - c^2}}{c}$.

(b) A bead is constrained to move on a smooth wire in the form of an equiangular spiral. It is attracted to the pole of the spiral by a force $m\mu(\text{distance})^2$ and starts from rest at a distance b from the pole. Show that, if the equation of the spiral $be\ r = ae^{\theta\cot\alpha}$ the time arriving at the pole is $\frac{\pi}{2}\sqrt{\frac{b^3}{2\mu}\sec\alpha}$. 7+8

6. (a) A heavy bead slides on a smooth fixed vertical circular wire of radius a. It it be projected from the lowest point with velocity just sufficient to carry it to the hights point, show that the radius to the bead at time t inclined to the vertical at an angle $2\tan^{-1}\left\{\sinh\sqrt{\frac{g}{a}}\right\}$.

(b) A particle starts from the origin in the direction of the initial line with velocity $\frac{f}{\omega}$ and moves with constant angular velocity ω about the origin and with constant negative radial acceleration –f. Show that the rate of growth of the radial velocity is never positive, but tends to the limit zero. Also prove that the equation of the path is $\omega^2 r = f\left(1-e^{-\theta}\right)$. 8+7

7. (a) A particle of mass m moves in a straight line under an attractive force mn^2x towards a fixed point on the line, when at a distance x from it. If it is projected with a velocity V towards the centre of force from an initial distance a from it, them prove that it will reach the centre of the force after a time $\frac{1}{n}\tan^{-1}\left(\frac{na}{V}\right)$.

(b) Deduce the expressions for radial and cross-radial velocity and acceleration of a particle moving in a plane in polar co-ordinates. 7+8

8. (a) A particle is projected with velocity u at an angle a to the horizon under gravity. Find the equation of the path of the particle and time of flight.

(b) If the distance x of a point moving in a straight line at any time t is given by $x = a\cos kt + b\sin kt$, where a and b are constants; Show that the motion of the point is simple harmonic. If $a = 3, b = 4$ and $k = 4$, then find the period, amplitude and maximum velocity. 7+8

Or

LINEAR PROGRAMMING

Answer any ***four*** from the following questions : 15×4=60

1. (a) At a cattle breeding firm, it is prescribed that the food ration for one animal must contain at least 14, 22 and 11 units of nutrients A, B and C respectively. Two different kinds of foder are available. Each unit weight of these two contains the following amounts of the three nutrients

	Fodder 1	Fodder 2
Nutrient 1	2	1
Nutrient 2	3	3
Nutrient 3	1	1

It is given that the cost of fodder 1 and fodder 2 are 3 and 2 monetary units respectively. Formulate the problem of finding the minimum cost of the purchasing the fodder as L.P.P. 8

(b) How many basic solutions are there for the following set of equations ? Find all of them.

$$x_1 + 3x_2 + 2x_3 + 3x_4 = 10$$
$$2x_1 - 3x_2 + 4x_3 + 6x_4 = 16$$

7

2. (a) Solve graphically the L. P. P.

$$Maximize,\ Z = 5x_1 - 2x_2$$
$$s.t.\ \ 5x_1 + 6x \geq 30$$
$$9x_1 - 2x_2 = 72$$
$$x_2 \leq 9$$
$$x_1 \geq 0, x_2 \geq 0$$

8

(b) Prove that convex polyhedron is a convex set. 7

3. (a) Prove that in E^2, the set $X = \{(x, y) : y^2 \le x\}$. 7

(b) Prove that if a L. P. P. has two optimal feasible solutions, then there are infinite number of optimal solutions. 8

4. (a) Solve the L. P. P. by simplex method

$$\begin{aligned} &\textit{Maximize. } Z = 4x_1 + 3x_2 \\ &\quad s.t.\ 3x_1 + x_2 \le 15 \\ &\quad\quad 3x_1 + 4x_2 \le 24 \\ &\quad\quad x_1 \ge 0,\ x_2 \ge 0 \end{aligned}$$

8

(b) Prove that any point of a convex polyhedron can be expressed as a convex combination of its extreme points. 7

5. (a) Slove the L. P. P. by simplex method

$$\begin{aligned} &\text{Maximize, } Z = x_1 + x_2 + 3x_3 \\ &\quad \text{s.t. } x_1 + 2x_2 - x_3 \le 10, \\ &\quad\quad 3x_2 + 2x_3 \le 8, \\ &\quad\quad x_1 + 3x_2 \le 15, \\ &\quad\quad x_1, x_2, x_3 \ge 0 \end{aligned}$$

8

(b) Prove that the set of all feasible solutions to a L. P. P. $Ax \ge b, x \ge 0$ is a convex set. 7

6. (a) Solve the L. P. P. by Big-M method

$$\begin{aligned} &\text{Maximize, } Z = 4x_1 + 3x_2 \\ &\quad \text{s.t.}\quad x_1 + 2x_2 \ge 8, \\ &\quad\quad 3x_1 + 2x_2 \ge 12, \\ &\quad\quad x_1, x_2 \ge 0 \end{aligned}$$

8

(b) Write the dual of following problem.

Manimize, $Z = 2x_1 + 3x_2 + 4x_3$

s.t. $x_1 + x_2 + x_3 \geq 4,$

$2x_1 - x_2 \leq -1,$

$2x_2 - 3x_3 = 2,$

$x_{1,} x_2 \geq 0$ and x_3 is unrestricted in sign. 7

7. (a) Solve the L. P. P. by Two Phase method

Maximize, $Z = 5x_1 + 11x_2$

s.t. $2x_1 + x_2 + \leq 4,$

$3x_1 + 4x_2 \geq 24,$

$2x_1 - 3x_3 \geq 6,$ 8

$x_1, x_2 \geq 0$

(b) Prove that dual of the dual is primal itself. 7

8. (a) Slove :

Manimize, $Z = 3x_1 + 2x_2$ 8.

s.t. $2x_1 + x_2 + \geq 14,$

$x_1 - x_2 \geq 4,$

$x_1, x_2 \geq 0$

(b) Prove that the optimal hyperplane is a supporting hyperplane in convex set of feasible solutions of a L. P. P. 7

Or

NUMERICAL METHODS

Answer any ***four*** from the following questions : 15×4=60

1. Establish Lagrange's interpolation formula for a set of $(n+1)$ points that the Lagrangian function are invariant under a linear transformation of the independent variable. Outline its merits and demerits as compared to other interpolation formula ? 15

2. Describe Runge-Kutta method of second and forth order for solving first order initial value problem. Use the above methods to find y at $x=0.02$ by taking $h=0.02$,

$$\frac{dy}{dx}=x^2+y,\ y(0)=1$$ 15

3. Establish Newton-Cotes's formula for numerical integration (errof is not required). Hence deduce Composite Trapezoidal rule form this formula and explain the geometrical interpretaion of trapezodial rule. Also calculate the error term of this formula. 15

4. Explain Gauss-Seidel method for solving a system of linear equations. When does the method fail ? Solve the following system of equations by Gauss elimination method

$$x_1+x_2+x_3=4$$
$$2x_1=x_2+3x_3=1$$
$$3x_1+2x_2=x_3=1$$ 15

5. Explain the principle of numerical differentiation. Given a table of values of $f(x)$ for $x=x_r=x_0+rh(h>0);\ r=0,1,2,\ldots,n;$ find the formula for numerical computation of $f'(x)$ and $f''(x)$ for $x=x_0$. 15

6. Define the degree of precision of the quadrature formula. What are the degrees of precision for (i) Trapizoidal and (ii) Simpson's 1/3 rules ? Evaluate the integral $\int_0^2 (4x-3x^2)dx$ taking $n=10$ by using Trapezodial and Simpsons rule and compare with its exact solution. 15

7. Explain the Newton-Raphson method for computing the real root of an equation $f(x)=0$ and its geometrical significance. Show that Newton-Raphson process has quardratic convergence. Find the condition of convergence. Write advantages and disadvantages of this method. 15

8. Define ill-conditioned and well conditioned problem. Describe LU-factorization method and hence solve the following system of equations

$$x-5y+z=2$$
$$2x+4y+z=1$$
$$x+y+z=0$$

15

Or

INTEGER PROGRAMMING AND THEORY OF GAMES

Answer any ***four*** from the following questions : 15×4=60

1. (a) What is integer programming problem ? Explain the merits and demerits of 'rounding-off' a optimal solution to an linear programming problem in order to obtain an integer solution.

(b) A machine shop has one lathe and four milling machines which are to be used to produce an assembly consisting of two parts 1 and 2. The production time in minute per parts is given below.

Part	Lath	Millling Machine
1	3	16
2	5	12

It is desired to maintain a balanced loading on all machines such that no machine runs more than 30 minutes per day longer than any other machine. The objective is to derive work time of each milling machine evenly to be obtained the maximum number of completed assemblies assuming an 8 hour working day. Formulate the integer programming porblem. 2+5+8

2. (a) Find the ranges of p and q which will render the entry (2,2) a saddle point for the game.

Player B

		B_1	B_2	B_3
	A_1	2	4	5
Player A	A_2	10	7	q
	A_3	4	p	6

(b) Slove the following integer programming problem using branch and bound method

Maximize $z = 5x_1 + 4x_2$

Subject to
$x_1 + x_2 \le 5$
$10x_1 + 6x_2 \le 45$
$x_1, x_2 \ge 0$ and integers

5+10

3. (a) Explain the steps of construction of Gomory's constraint of an integer programming problem.

(b) Derive the first Gomory's constraint of the following integer programming problem

Maximize $z = x_1 + 2x_2$

Subject to
$2x_2 \le 7$
$x_1 + x_2 \le 7$
$2x_1 \le 11$
$x_1, x_2 \ge 0$ and integers

7+8

4. (a) What are the main characteristics of games ?

(b) Solve the following integer programming problem using Gomory's cutting plane method

Maximize $z = 2x_1 + 3x_2$

Subject to
$5x_1 + 4x \le 18$
$x_1, x_2 \ge 0$ and integers

5+10

5. (a) Explan the methods of dominance in the solution of rectangular games.

(b) Reduce the following game by dominance method and find the game value :

Player B

Player A		*I*	*II*	*III*	*IV*
	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

6+9

6. Solve the following game by graphical method

		Player B			
		y_1	y_2	y_3	y_4
Player A	x_1	19	6	7	5
	x_2	7	3	14	6
	x_3	12	8	18	4
	x_4	8	7	13	– 1

6+9

7. Formulate the following game as a linear programming problem and hence solve it.

		B		
		1	2	3
A	1	3	– 4	2
	2	1	– 3	– 7
	3	– 2	4	7

6+9

8. (a) Explain the following : (i) Minimax and Maximin principles (ii) Pure and mixed strategies (iii) Two-person zero-sum game.

(b) For a two-person zero-sum game, the payoff matrix for player A is $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with no saddle point. Obtain the optimal strategies (x_1, x_2) and (y_1, y_2). 3+3+3+6