

2021

MATHEMATICS

[Honours]

PAPER –VII

Full Marks : 90

Time : 4 hours

*The figures in the right-hand margin indicate marks**Candidates are required to give their answers in their own words
as far as practicable**Illustrate the answers wherever necessary*

GROUP – A

Answer any **one** of the following : 10 × 1

1. Describe with the help of a block diagram the operations of different components of a computer. 10
2. If X is normal (m, σ) distribution, then prove that

$$P(a < x < b) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right) \text{ and } P(|x - m| > a\sigma) = 2\{1 - \Phi(a)\}$$

where $\Phi(x)$ denotes the standard normal distribution function. 10

GROUP – B

Answer any **four** of the following : 20 × 4

3. (a) Write short notes on Source program, Object program and Compiler. 10
- (b) Define and explain the following : 10
 - (i) Binary half-adder
 - (ii) Full-adder.
4. (a) If A, B, C are the inputs, obtain the circuit to get the output of the function F , where

$$F = AC' + A'(BC' + B'C) \quad \text{10}$$

(Turn Over)

- (b) Subtract binary numbers 1010 and 111001 using 2's complement. 5
- (c) Multiply the binary numbers 1001 and 1101. 5
5. (a) Let X be a standard normal variate. Show that $Y = \frac{1}{2}X^2$ is $\gamma\left(\frac{1}{2}\right)$ variate. 10
- (b) What is the probability of getting an odd sum when two fair dice are thrown. 10
6. (a) If X and Y are two jointly distributed discrete random variables then prove that
- $$E(X+Y) = E(X) + E(Y). \quad 10$$
- (b) If (X, Y) has the normal distribution in two dimensions with zero means, unit variances and correlation coefficient ρ , then prove that the expectation of the greater of X and Y is $\sqrt{(1-\rho)/\pi}$. 10
7. (a) Define moment generating function of a random variable X . A continuous distribution has probability density function $f(x) = a$. Obtain the moment generating function. 10
- (b) Let the random variables X_1, X_2, \dots, X_n be independently and identically distributed each assuming values 0 and 1 with probabilities p and $(1-p)$ respectively. Derive Bernoulli's law of large numbers in this case. 10
8. (a) What is meant by a statistical hypothesis? What are the two types of error of decision committed in testing of statistical hypothesis? Explain the concept of critical region. 10
- (b) The joint density function of the random variables X and Y is given by :
- $$f(X, Y) = X + Y, \quad 0 < X < 1, \quad 0 < Y < 1$$
- $$= 0, \quad \text{elsewhere}$$
- Find the distribution of XY . 10
9. (a) Define 'Likelihood function'. Explain the method of maximum likelihood in parameter estimate. 10
- (b) What is meant by asymptotically normal distribution? If X_n is a binomial (n, p) variate, show that
- $$\frac{X_n - np}{\sqrt{npq}}$$
- is asymptotically normal $(0, 1)$. 10

10. (a) Find a confidence interval for the mean m of a normal population on the basis of a sample of size n having confidence co-efficient is $1 - \epsilon$. 10

(b) Given a sample of size n form a normal (m, σ) population, discuss how to find out a symmetrical 95% confidence interval for the mean when σ is known.

$$\left[\frac{1}{\sqrt{2\pi}} \int_0^{1.96} e^{-t^2/2} dt = 0.4750 \right] \quad 10$$
