2021

MATHEMATICS

[Honours]

PAPER – VIII

Full Marks : 90

Time : 4 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

(*Numerical Analysis*)

[*Marks* : 25]

1. Answer any *one* question : 5 × 1

(*a*) Write down the quadratic polynomial which takes the same value as $f(x)$ at $x = -1, 0, 1$ and integrate it to obtain the integration rule

$$\int_{-1}^{1} f(x)dx \cong \frac{1}{3}[f(-1)+4f(0)+f(1)].$$ 5

(*b*) Use Euler-Maclaurin sum formula to prove the formula :

$$\sum_{i=1}^{n} x^3 = \frac{n^2(n+1)^2}{4}.$$ 5

2. Answer any *one* question : 20 × 1

(*a*) (*i*) Explain the principle of numerical differentiation. Deduce Simpson's 1/3rd rule of numerical integration. Deduce the degree of precision of Simpson's 1/3rd rule. 2 + 5 + 3

(*ii*) What do you mean by round-off error and truncation error ? Explain with example. Discuss how round-off error propagates in a difference table. 5 + 5

(*Turn Over*)

(*b*) (*i*) Define the order of convergence of an iterative process. Discuss the Newton Raphson method for finding a real root of an equation $f(x) = 0$ and determine its order of convergence. 2 + 4 + 4

(*ii*) Discuss the Euler's method of solving a first order differential equation. 4

(*iii*) Deduce the formula

$$\frac{d}{dx} f[x, x, \ldots, r \text{ times } x] = rf[x, x, \ldots (r+1) \text{ times } x] \text{ and}$$

$$\frac{d^r}{dx^r} f[x] = r!\, f[x, x, \ldots, (r+1) \text{ times } x],$$

where the symbols have their usual meanings. 3

(*iv*) Using infinite series expansion of $\sin h(x)$, compute $\sin h(1.4)$, correct up to four decimal places. 3

GROUP – B

(*Real Analysis*-III)

[*Marks* : 25]

3. Answer any *one* question : 5 × 1

(*a*) Find the Fourier series of f where

$$f(x) = \begin{cases} 0 & \text{if } -\pi \le x \le 0 \\ \dfrac{\pi x}{4} & \text{if } 0 \le x \le \pi \end{cases}$$

Hence show that the sum of the series $1 + \frac{2}{3^2} + \frac{2}{5^2} + \cdots$ is $\frac{\pi^2}{8}$. 5

(*b*) Assuming the power series expansion for $\frac{1}{\sqrt{1-x^2}}$ as

$$\frac{1}{\sqrt{1-x^2}} = 1 + \frac{1}{2}x^2 + \frac{1\cdot 3}{2\cdot 4}x^4 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^6 + \cdots.$$ 5

4. Answer any *one* question : 20 × 1

(*a*) (*i*) Define point wise and uniform convergence of a sequence of functions defined on a set. Check the uniform and point wise convergence of the sequence of functions $\left\{\frac{nx}{1+n^2x^2}\right\}$. (2 + 4) (*ii*) Let $D \subset \mathbb{R}$, and a sequence of functions $\{f_n\}$ be uniformly convergent on D to a function f. Let $x_0 \in D'$ (the derived set of D) and $\lim_{x \to x_0} f_n(x) = a_n$, then prove that the sequence $\{a_n\}$ is convergent and $\lim_{x \to x_0} f(x) = \lim n \to \infty an.$ 6

(*iii*) Let $I = [a, b]$ be a closed bounded interval and for each $n \in N, f_n : I \to \mathbb{R}$ be integrable on I to the function s then, prove that s is integrable on I and $\sum \int_a^b f_n(x)dx = \int_a^b s(x)dx.$ 6

(*iv*) Define radius of convergence of a power series. 2

(*b*) (*i*) State and prove Weierstrass M-test for the convergence of a series of functions. 6

(*ii*) Show that the series

$$\sum_{n=1}^{\infty} \frac{x}{n^p + x^2 n^q}$$

converges uniformly for all real x if $p + q > 2$. 6

(*iii*) Let

$$\sum_{n=1}^{\infty} a_n x^n$$

be a power series with radius of convergence R (> 0). Then prove that the series is uniformly convergent on $[-s, s]$, where $0 < s < R$. 6

(*iv*) When a function $f : [a, b] \to \mathbb{R}$ is said to satisfy Dirichlet's conditions ? 2

GROUP – C

(*Linear Algebra*-II)

[*Marks* : 10]

5. Answer any *one* question : 10 × 1

(*a*) (*i*) Define the rank and the nullity of a linear transformation. Let V and W be two subspaces of a finite dimensional vector space over a field F and V is finite dimensional. If $T : V \to W$ be a linear mapping then prove that :

The nullity of T + The rank of $T = \dim V$. 2 + 6 (*ii*) Define linear transformation on vector spaces and its kernel. 2

(*b*) (*i*) When a linear transform is said to be invertible ? If a linear transform is invertible then prove that the inverse transform is also linear. Let a linear transform $\mathbb{R}^3 \to \mathbb{R}^3$ is defined by

$$T(x, y, z) = (x - y, x + 2y, y + 3z), (x, y, z) \in \mathbb{R}^3 .$$

Show that T is invertible and determine T^{-1}. 1 + 3 + 4

(*ii*) Determine the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$, which maps the basis vectors (1,0,0), (0,1,0), (0,0,1) of $\mathbb{R}^3$ to the vectors (1,1), (2, 3), (3, 2) respectively. 2

GROUP – D

(Program must be written either in FORTRAN or C language)

(**Practical**)

[*Marks* : 30]

Answer any **two** questions : 15 × 2

6. Write an algorithm and a program to find the area and perimeter of a triangle whose three sides are given.

7. Write an algorithm and a program to find the maximum and minimum among n numbers.

8. Write an algorithm and a program to find G.C.D. between two integers.

9. Write an algorithm and a program to test a number prime or not.

10. Write an algorithm and a program to find the value of ${}^nP_r$ for given values of n and r.

11. Write an algorithm and a program to search a key number from a set of numbers.

12. Write an algorithm and a program to add a square matrix A to its transpose matrix.

13. Write an algorithm and a program to find the product of diagonal elements of a square matrix A.

14. Write an algorithm and a program which is convert uppercase characters of string to lowercase characters.

15. Write an algorithm and a program to count the number of words present in a string.

16. Write an algorithm and a program to find the value of sin (0·175) by Lagrange interpolation technique of the following information :

x	0·15	0·17	0·18	0·21	0·23
sin$x$	0·14944	0·16918	0·18886	0·20846	0·22798

17. Write an algorithm and a program to evaluate

$$\int_{1}^{1\cdot 8}(2x^{13}+\sin x)dx$$

by Simpson $\frac{1}{3}$rd rule taking 1000 sub-intervals.

18. Write an algorithm and a program to find a real root of the function $f(x) = x^3 + x + 1$ by fixed point iteration method, correct up to 4 decimal places.

19. Write an algorithm and a program to find the value of $y(0.2)$ from the differential equation

$$\frac{dy}{dx} = 1 + y\sin x - x^2, \quad x(0.0) = 0$$

by second order Runge-Kutta method.

20. Write an algorithm and a program to find median and mode for discrete distribution.

21. Write an algorithm and a program to calculate mean and standard deviation for the group frequency distribution.

Class Interval	0-5	5-10	10-15	15-20	20-25
Frequency	6	8	7	4	5

22. Write an algorithm and a program to fitting a straight line through a set of points (x_i, y_i). Demonstrate your program for the following information :

x	2	4	6	8	10
y	1·5	2·5	3·5	4·5	5·5