বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2020

(Under CBCS Pattern)

Semester - III

Subject: MATHEAMATICS

Paper: DSC 1C/2C/3C-T

(Real Analysis)

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer any ***three*** from the following questions : 3×20

1. (a) Consider the series $\sum_{n=1}^{\infty} 50\frac{\sin(nx)}{n(n+1)}$. Is this series uniformly convergent in any interval ? 4

(b) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ where $a_n = (-1)^n \frac{n^n}{n!2^n}$. 4

(c) Let $f_n(x) = x^n,\ x \in [0,1]$. Show that the sequence of functions $\{f_n\}$ is not uniformly convergent. 4

(d) If a power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $= x_1$, then prove that the series converges absoultely for all real x satisfying $|x| < |x_1|$. 4

(e) Prove that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is convergent. 4

2. (a) Prove that

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \frac{1}{\sqrt{n^2+3}} + \ldots + \frac{1}{\sqrt{n^2+n}}\right) = 1.$$ 10

(b) Show that the series $1 - \frac{1}{2} + \frac{1.3}{2.4} - \frac{1.3.5}{2.4.6} + \ldots$ is convergent. 10

3. (a) A sequence u_n is defined by $u_{n+2} = \frac{1}{u}(u_{n+1} + u_n)$ for $n \geq 1$ and $0 < u_1 < u_2$. Prove that the sequence $\{u_n\}$ converges to $\frac{u_1 + 2u_2}{3}$. 10

(b) Discuss the convergence of $\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots$ 10

4. (a) Let $f_n(x) = nx(1-x)^n,\ x \in [0,1]$ for each $n \in N$. Show thast the limit function f is continuous . But $< f_n(x) >$ does not converge to uniformly. 10

(b) Prove that the series of functions

$$\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1(3x+1)} + \ldots x \geq 0$$

is convergent on $[0, \infty)$. 10

5. (a) A function f is defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x) = 1 + 2.3x + 3.3^2.x^2 + \ldots + n.3^{x-1}.x^{n-1} + \ldots.$$

show that f is continuous on $\left(-\frac{1}{3}, \frac{1}{3}\right)$. Evaluate $\int_0^{\frac{1}{4}} f\,dx$. 10

(b) Find the radius of convergence of the power series

$$1 - \frac{2^2}{3^2}x + \frac{2^2 4^2}{3^2 5^2}x^2 - \frac{2^2 4^2 6^2}{3^2 5^2 7^2}x^3 + \ldots$$ 10

6. (a) Prove that the series $(1-x)^2 + x(1-x)^2 + x^2(1-x)^2 + \ldots$ is uniformly convergent on [0, 1]. 10

(b) Prove that a power series can be differentiated term by term within the interval of convergence. 10
