2021

MATHEMATICS

[Honours]

PAPER – VI

Full Marks : 100

Time : 4 hours

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

GROUP – A

Answer any **one** question : 10 × 1

1. Deduce the necessary and sufficient conditions for equilibrium of a fluid under the action of external force of components (X, Y, Z) per unit mass acting at the point (x, y, z) in the fluid. Is the condition true for irrotational field of force ? Justify. 10

2. Show that the equation of momental ellipsoid at a point on the rim of a hemisphere is $2x^2 + 7(y^2 + z^2) - \frac{15}{4}xz = \text{const.}$ 10

GROUP – B

Answer any **four** questions : 20 × 4

3. If a rigid body moves under the action of a system of conservative forces, then show that the sum of its kinetic and potential energies remain constant throughout its motion. 20

4. A uniform vertical circular plate, of radius a is capable of revolving about a smooth horizontal axis through its centre. A rough perfectly flexible chain, whose mass is equal to the mass of the plate whose length is equal to its circumference, hangs over its rim in equilibrium. If one end be slightly displaced, then show that the velocity of the chain when the other end reaches the plate is $\sqrt{\frac{\pi ag}{6}}$. 20

5. Deduce the formula $HM = \frac{AK^2}{V}$, the symbols have their usual meaning. Find also the metacentric height of a ship for rolling displacements. 20

6. (*i*) Prove that a non-empty connected graph G is Eulerian iff its vertices are all of even degree. 10

(*ii*) Define tree and spanning tree. Prove that an undirected graph is a tree, if and only if, there is a unique simple path between every pair of vertices. 2 + 2 + 6

7. Using the generating functions solve the recurrence relation :

$$a_n = 5a_{n-1} - 6a_{n-2},$$

for all $n \geq 2$ with initial conditions $a_0 = 6$ and $a_1 = 30$. 20

8. The differential equation

$$\frac{dP}{dt} = \frac{1}{1000}(1500 - P)P,$$

describes the growth and regulation of a fish population in an empty region of sea. At time $t = 0$, there are $P(0) = 2000$ fish. Find P as a function of t, sketch the graph of this function. 20

9. A uniform rod is held at an angle α to the horizon with one end in contact with a horizontal table whose coefficient of friction is μ, if it be then released show that it will commence to slide if $\mu < \frac{3ta}{1+4\tan^2\alpha}$. 20

10. A circular disc of radius 'a' is just completely immersed with its plane verticle in a homogeneous liquid. Prove that the distance between the centres of pressure of the two semi-circles into which the disc is divided by its horizontal diameter is $\frac{18\pi a}{9\pi^2 - 16}$. 20

[*Internal Assessment* : 10 Marks]