বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

B.Sc. General Examination 2021

(CBCS)

4th Semester

MATHEMATICS

PAPER—SEC2T

Full Marks : 40

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

SEC2T : GRAPH THEORY

Answer any *two* questions. 2×15

1. (a) Show that sum of degree of vertices of a graph is equal to twice the number of edges. Hence show that number of odd degree vertices of a graph must be even.

(b) Show that a non-null graph is bipartite if and only if it does not contain any cycle of odd degree.

(c) Let G be a graph. Eigen values of G are the eigenvalues of the adjacency matrix A_G. Also, let $V(G)$ = $\{v_1, v_2, \ldots\ldots\ldots, v_n\}$, $A_G = (a_{ij})_{m \times n}$ and $\Delta(G) = \max\{\deg(v) / v \in V(G)\}$. Then show that if λ be an eigenvalue of A_G, $|\lambda| \le (G)$. 5+5+5

2. (a) Define circuit of a graph. Prove that a circuit free graph with n vertices and $(n-1)$ edges is a tree.

(b) Prove that $K_{3,3}$ and K_5 are not planer.

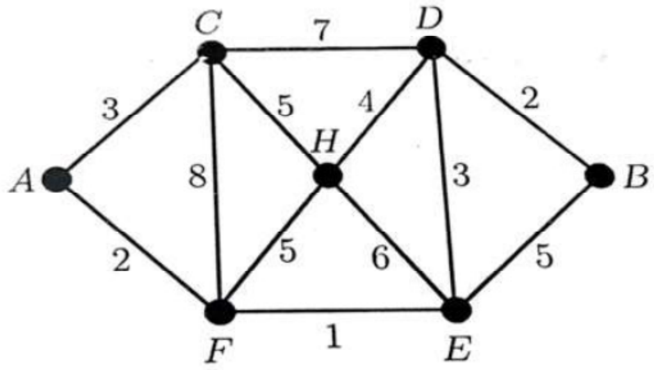
(c) Show that a simple graph with n vertices and m components can have at most $\frac{1}{2}(n-m)(n-m+1)$ edges. 5+5+5

3. (a) Show that any connected graph with n vertices e edges and f faces satisfy the equation $(n - e + f) = 2$.

(b) Show that a graph with $2n$ vertices, if the degree of each vertex at least n then show that the graph is connected.

(c) Let G be a graph with 30 vertices such that for any two vertices u and v of G and $\deg(u) + \deg(v) \ge 29$. Prove that G is connected. 5+5+5

4. (a) Let G be a connected graph with number of edges 11. Find the maximum possible vertices of G.

(b) 

Above figure shows the distance among the cities. Find the shortest distance from the vertex A to the vertex B, using Dijkstra's algorithm.

Answer any *one* question. 1×10

5. Prove that if the degree of each vertex of a graph is of even then the graph has an Euler circuit. 10

6. Define Hamiltonian circuit and give an example of graph which is Hamiltonian but not Eulerian. Define diameter, eccentricity and radius of a graph. Show that radius of tree T is equal to [diam(T)/2]. 10

SEC2T : INTEGRAL CALCULUS

Answer any *two* questions. 2×15

1. (a) Evaluate the following integrals :

(i) $\int \frac{x^2}{x^4 + x^2 - 2} dx$

(ii) $\int \frac{dx}{\left(4-3x^2\right)\sqrt{3+4x^2}}$

(b) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx, \ \left(n \in \mathbb{Z}^+, n > 1\right),$ *show that* $I_{n+1} - I_{n-1} = \frac{1}{n}$. Hence evaluate

$\int_0^{\frac{\pi}{4}} \tan^9 \tan^9 x dx$. 4+4+(4+3)

2. (a) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log \log 2$.

(b) If $f(x) = f(a+b-x)$, then show that $\int_a^b x f(x) dx = \frac{a+b}{2} \int_a^b f(x) dx$.

(c) Find the area bounded by the parabolas $x^2 = 4y$ and $y^2 = 4x$.

5+5+5

3. (a) Find the total length of the astroid : $x^{(2/3)} + y^{(2/3)} = a^{(2/3)}$.

(b) Find the volume of the solid generated by revolving the cardioide $r = a\ (1 - \cos\theta)$ about the initial line.

(c) Show that the surface area of the solid generated by revolving the cycloid

$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about the line $y = 0$ is $\frac{64}{3}\pi a^2$.

5+5+5

4. (a) Evaluate $\iint \frac{\sqrt{a^2b^2 - b^2x^2 - a^2y^2}}{\sqrt{a^2b^2 + b^2x^2 + a^2y^2}} dx\, dy$, over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Find the Volume of an ellipsoid of revolution about major axis.

8+7

Answer any *one* question. 1×10

5. (a) Evaluate : $\int \frac{dx}{(5 + 4\cos x)^2}$.

(b) Show that $\iint e^{\frac{y-x}{y+x}} dx\, dy$ taken over the triangle with vertices at (0,0), (0,1) and (1,0) is $\frac{1}{4}\left(e - e^{-1}\right)$. 5+5

6. (a) Obtain a reduction formula for $\int_0^1 x^m (1-x)^n\, dx, \left(m, n \in \mathbb{Z}^+\right)$. Hence deduce its value.

(b) Show that the length of the arc of the parabola $\frac{l}{r} = 1 + \cos\cos\theta$ cut off its latus rectum is $l\{\sqrt{2} + \log(1+\sqrt{2})\}$. 5+5

SEC2T : MATHEMATICAL FINANCE

Answer any *two* questions. 2×15

1. (a) Discuss briefly the concept of "Time Value of Money".

(b) Distinguish between NPV and IRR. 7+8

2. (a) Define floating-rate bonds.

(b) State the process of calculating expected return of a portfolio. 5+10

3. Discuss briefly the theorems on bond pricing. 15

4. Define short selling. State the procedure of short selling. 15

Answer any *one* question. 1×10

5. Discuss the procedure of calculating simple interest and compound interest.

6. Write short notes on : 5+5

(a) Asset Return.

(b) Inflation.

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