বিদ্যাসাগর বিশ্ববিদ্যালয়
VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examination 2021
(Under CBCS Pattern)
Semester - VI
Subject: MATHEMATICS
Paper: SEC 4-T & P

Full Marks : 40
Time : 2 Hours

Candiates are required to give their answer in their own words as far as practicable.
The figures in the margin indicate full marks.

PROBABILITY AND STATISTICS

Group A

Answer any ***two*** from the following questions : 15×2=30

1. (i) The joint probability density function of the random variables X and Y is

$$f(x, y) = \begin{cases} 8xy, & 0 \le x <= y, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Examine whether X and Y are independent. Also compute Var (X) and Var (Y). 5

(ii) From a pack of 52 cards an even number of cards is drawn. What is the probability case. The probability that Ram will speak the truth is x a nd the probability that Rahim

will speak the truth is y. Ram and Rahim agree in a certain statement. What is the probability that this statement is true ? 5

2. (i) If m and μ_r denote the mean and r-th order central moment of a Poisson – distribution, then prove that $\mu_{r+1} = rm\mu_{r-1} + m\frac{d\mu_r}{dm}$. Hence find the standard deviation. 5

(ii) A problem in Mathematics is given to $(n-1)$ students whose chances of solving it are respectively $\frac{1}{2}, \frac{1}{3}, \cdots \frac{1}{n}$. What is the probability that the problem will be solved ? 5

(iii) The train services from Howrah to Midnapore has the frequency within 10 minutes. You reach in a station to avail the said route. Find the porbability that (a) to wait at least 5 minutes, (b) at most 6 minutes and (c) the time to wait lying between 4 to 7 minutes. 5

3. (i) Give axiomatic definition of probability and show that it statisfies the conditional probability. 6

(ii) Define characteristic function of a random variable X. Find the characteristic function of Noraml (m, σ). 9

4. (i) The random variable X has the distribution given by $P(X=k) = 2^{-k}$, $k = 1, 2, ...$Show that $E(X) = 2$, Var $(X) = 2$. 5

(ii) If $\sigma_x^2, \sigma_{x-y}^2$ be the variances of X and Y and $X - Y$ respectively. Then prove that ρ_{xy} (co-relation co-efficient between x and y) $= \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$. 5

(iii) In a binomial (8, p)- distribution $P(X=2) = P(X=3)$. Find $P(X=0)$ and $P(X=8)$. 5

Group B

Answer any ***one*** from the following questions : 1×10=10

5. (i) Explain statistical regularity in a sequence of trials and hence give the frequency definition of probabilty. 5

(ii) If a dice is thrown n times. Shaw that the probability of a even number of sixes is $\frac{1}{2}\left(1+\frac{2^n}{3^n}\right)$. 5

6. (i) Consider the probability density function of a random variable X is given by $f(x)=ae^{-b|x|}, x\in(-\infty,\infty)$. Find cumulative distribution function and the relation between a and b. 5

(ii) Show that for any two arbitrary events A and B,

$$Max\left[P(A),P(B)\right]\le P(A\cup B)\le Min\left[P(A)+P(B),1\right].$$ 5

Or

FORECASTING

Group A

Answer any ***two*** from the following questions : 15×2=30

1. Describe the types of forecasts by time horizon, and write the differences between them. 15

2. Discuss qualitative methods of forecasting in brief with proper examples. 15

3. What is a time series and what are its uses ? What are the various methods for dertermining trend in a time series ? Define secular trend of a time series and explain methods that are used in isolating it. 2+3+10

4. Explain the method of moving average for the determination of trend. What are the advantages and disadvantages of this method ? 10+5

Group B

Answer any ***one*** from the following questions : 10×1=10

5. What is forecasting ? What is the necessity of forecasting ? Discuss the importance of forecasting in different fields. What are the steps of forecasting ? 2+2+4+2

6. Calculate seasonal variation for the following data of sale in thousands Rs. of a firm by any method. 10

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
1979	30	40	36	34
1980	34	52	50	44
1981	40	58	54	48
1982	52	76	68	62

Or

PORTFOLIO OPTIMIZATION

Group A

Answer any ***two*** from the following questions : 15×2=30

1. (a) State the characteristics of financial market. 8

   (b) Stat the objectives of investment. 7

2. (a) Discuss briefly the different types of risk. 7

   (b) State the procedure for the measurement of systematic risk. 8

3. How portfolio risk can be reduced through diver sification ? — Discuss briefly. 15

4. Discuss briefly the Markowitz model of portfolio theory. 15

Group B

Answer any ***one*** from the following questions : 10×1=10

5. Distinguish between business risk and financial risk. 10

6. Write short notes on :

   (a) Risk-face Assets. 5

   (b) Efficient Frontier. 5

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Or

UNDERSTANDING PROBABILITY AND STATISTICS THROUGH (PRACTICAL)

Group A

Answer any ***two*** from the following questions : 15×2=30

1. (a) Draw a random sample of 6 boys, without replacement, from 140 boys of a College in Medinipur with the help of random sampling numbers.

   (b) The heights in inches of eight college students chosen at random were as follows :

   62.2, 62.4, 63.1, 63.2, 65.5, 66.2, 66.3, 66.5

   Find the 95% and 98% confidence intervals for SD of the population of heights. 8+7

2. (a) A survey of 400 families with 5 children each revealed the following distribution :

No. of Boys	0	1	2	3	4	5
No. of Families	30	60	100	140	54	16

   Fit a binomial distribution to the data and find the expected frequencies for different classes. Also comment on goodness of it.

   (b) Assuming that half the population is consumer of tea and each of 100 investigators takes a sample of 10 to see whether they are consumers of tea, how many investigators would you expect to report that 3 people or less are consumers of tea ? 8+7

3. (a) If the weekly wages of 10,000 workers in a factory follow normal distribution with mean and standard deviation Rs. 45 and Rs. 5 respectively find the expected numbers of workers whose weekly wages are (i) between Rs. 43 and Rs. 49, (ii) less than Rs. 43 and (iii) more than Rs. 49. Given that

   $\Phi(0.8) = 0.7881446, \Phi(0.4) = 0.6554217$.

   (b) The local authority in a certain city installed 2000 electric lamps in the streets of the city. The lamp life, meaured in hours, follow a normal distribution with average 1000 hours and standard deviation 200 hours.

(i) What number of lamps do you expect to fail in the first 700 hours ?

(ii) After what period of hours 10% of the lamps would be still burning ?

Give that $\int_0^x \frac{1}{\sqrt{2\pi}} e - \frac{t^2}{2} dt = 0.4000000, 0.4192433, 0.4331928$ when x=1.28, 140 & 1.50 respectively. 8+7

4. (a) The probability that an individual will suffer a bad reaction from a particular injection is e 0.001. Determine, by using the binomial and the poisson distribution the probability that out of 2000 individuals more than 2 individuals will suffer a bad reaction.

(b) Find a 95% confidence interval for the mean of a $N(m, \sigma)$ population using the following data $\bar{x} = 48, \sigma = 9, n = 36$.

$$P\left[\left(\frac{\sqrt{n(\overline{x-m})}}{\sigma}\right) \geq 1.960\right] = 0.025$$ 8+7

Group B

Answer any ***one*** from the following questions : 10×1=10

5. Height (in inches) and weights (in kg) of 5 persons are given below :

Height	64	60	67	59	69
Weight	57	60	73	62	68

(i) Determine the correlation coefficient between height and weight.

(ii) If by a defect in weighing machine, weight are recorded by 2 kg. more than the true weight, then find the value of the correct correlation coefficient.

(iii) Obtain the regerssion lines, ignoring (ii) and estimate the weight of a person having height 63 inches and height of a person having weight 66 kg. 10

6. Eight competitors in a musical contest were ranked by two judges A and B in the following manner. Calculate Spearman's rank correlation coefficient.

Sl no. of candidate	1	2	3	4	5	6	7	8
Judge A	6	8	2	5	3	4	1	7
Judge B	8	7	4	3	1	2	5	6

10
