Government General Degree College, Dantan-II

Internal Assessment/4th Semester/Session : 2019-2020

DSC-1D(Core-10)

Subject-**Mathematics**(General)

Full marks-30

Answer **any six** questions: $6 \times 5 = 30$

1.Let $(G,\circ)$ be a group. Prove that

(i) Identity element in G is unique, (ii)Each element in G has unique inverse.

2.In a group $(G,\circ)$ prove that (i) $(a^{-1})^{-1} = a$, for all $a \in G$ and $(a \circ b)^{-1} = b^{-1} \circ a^{-1}, \forall a, b \in G$.

3.Examine whether $(\mathbb{Z},\circ)$ is a group or not where $\circ$ is as $a \circ b = a + b + 1$, $\forall a, b \in \mathbb{Z}$.

4.Verify the group properties by taking the set of all permutations of the elements 1,2,3. Show further that the symmetric group S_3on $\{1,2,3\}$ is non-commutative.

5.If the elements a, b and ab in a group be each of order 2 , then prove that $ab = ba$.

6.Show that a non empty subset S of a group $(G,\circ)$ is a subgroup of $(G,\circ)$ if and only

if $a \circ b^{-1} \in S$, for all $a, b \in S$.

7.Show that the set of all fourth root of unity , namely $\{1, -1, i, -i\}$, forms an abelian group with respect to multuipication.

8.Define divisors of zero in a ring. If a be unit in a ring R, then show that a is not a divisor of zero.

9.If R be a ring such that $a^2 = a$, $\forall a \in R$ then prove that

$(i) a + a = 0$, for all $a \in R$.

$(ii) a + b = 0 \Rightarrow a = b$, $\forall a, b \in R$.